INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, First Semester, 2011-12
Statistics - IV, Final Examination, November 28, 2011

Answer any 5 questions. Maximum possible score is 50.

1. Suppose we have a random sample $X_1, \ldots, X_n$ from a continuous distribution with c.d.f. F and density f, both of which are completely unknown.
(a) Define the histogram estimate of f.
(b) Show that the histogram is a consistent estimator of f if the interval width is chosen to be proportional $n^{-2/3}$. [10]

2. Suppose $X_1, \ldots, X_n$ are i.i.d. Bernoulli(θ) where $0 < \theta < 1$ is unknown but n is fixed. Consider estimating θ under the loss
$L(\theta, a) = \theta^{-1/2}(1-\theta)^{-1/2}(\theta - a)^2$, $0 \le a \le 1$.
Let $\delta^*(X_1, \ldots, X_n)$ be any randomized estimator of θ. Prove that there exists a non-randomized estimator of θ which is at least as good as δ^*. [10]

3. Suppose $X_1, \ldots, X_n$ is a random sample from Poisson(θ), where $\theta > 0$. Consider the loss $L(\theta, a) = (\theta - a)^2/\theta$, where $a \ge 0$. Derive the minimax estimator of θ. [10]

4. Let $X_1, \ldots, X_n$ be a random sample from the exponential distribution with mean θ, where $\theta > 1$ is unknown. Consider the decision problem where the loss function is $L(\theta, a) = (\theta - a)^2$, $a \ge 0$. Show that the decision rule $\delta(X_1, \ldots, X_n) = \bar{X}$ is inadmissible as an estimator of θ. [10]

5. Solve the 2-person, zero-sum game with the following loss matrix:

	a_1	a_2	a_3	a_4
θ_1	4	4	0	0
θ_2	0	3	3	1
θ_3	0	2	0	2

[10]

6. Consider the 2-person, zero-sum game where player I chooses a number $\theta \in \Theta = [0, 1]$. Player II guesses this to be a number $a \in \mathcal{A} = [0, 1]$. The loss then (to player II) is $L(\theta, a) = (\theta - a)^2$. Find the minimax and maximin strategies. [10]